TRANSBOUNDARY CAPITAL AND POLLUTION FLOWS WITH ENDOGENOUS WEALTH, KNOWLEDGE AND ENVIRONMENT ACCUMULATION

Prof. Wei-Bin Zhang
Ritsumeikan Asia Pacific University
Jumonjibaru, Beppu-Shi, Oita-ken 874-8577 Japan
wbz1@apu.ac.jp

ABSTRACT

The paper deals with global dynamic interdependence between inequalities in income and wealth, economic structures, knowledge growth, capital accumulation, and environmental changes. It builds a trade model with endogenous wealth, environment, knowledge accumulation and labor and capital distribution between sectors and between countries under perfectly competitive markets and free trade. The model is built on the basis of the Solow-Uzawa neoclassical growth model, the Oniki-Uzawa trade model, some neoclassical growth models with environment, ideas about transboundary pollution in environmental economics, Arrow's learning-by-doing model, and Zhang's idea about knowledge as international public stock. We integrate these approaches by applying the utility function proposed by Zhang. Zhang's approach makes it analytically easier (than the traditional Ramsey approach) to analyze behavior of households with endogenous saving and consumption. We show that the dynamics of -country world economy is controlled by differential equations. We simulate the motion of the model with three countries and carry out comparative dynamic analysis with regard to some parameters. Our comparative dynamic analysis demonstrates how the global economies react to changes in knowledge utilization efficiently, the return to scale effect in knowledge accumulation, the tax rate on the capital good sector, the transboundary pollution, the tax rate on consumption, the population, and the propensity to save in the transitory processes as well as in the long term.

Keywords: *transboundary pollution; global economic growth; free trade; wealth accumulation; knowledge accumulation*

JEL Classification: F11, O44, F64

ABSTRAK

Tulisan ini mengupas dinamika interdependensi global antara ketimpangan dalam pendapatan dan kekayaan, struktur ekonomi, petumbuhan pengetahuan, akumulasi modal, dan perubahan lingkungan. Hal ini berguna untuk membangun model perdagangan dengan variable endogen seperti kekayaan, lingkungan, akumulasi modal, tenaga kerja, dan distribusi modal antar sector dan antar negara dibawah koridor pasar persaingan sempurna dan perdagangan bebas. Model ini dibentuk berdasarkan model pertumbuhan neoklasik dari Solow-Uzawa, model perdagangan Oniki-Uzawa merupakan model yang didasari dari beberapa model pertumbuhan dengan mempertimbangkan lingkungan dari kelompok neo-klasik. Ide-ide dasar seperti lintas batas polusi yang diajarkan dalam ekonomi lingkungann seperti model learning-by-doingnya Arrow, dan pengetahuan sebagai cadangan public internasional dari Zhang's. Kami mengintegrasian pendekatan-pendekatan ini dengan mengaplikasikan fungsi utilitas yang ditawarkan oleh Zhang. Pendekatan Zhang membuat analisis ini lebih mudah dipahami daripada yang dilakukan oleh Ramsey untuk analisa perilaku rumah tangga yang memasukan variabel endogen tabungan dan konsumsi. Model yang kami susun menunjukkan bahwa dinamika perekonomian di kontrol oleh persamaan diferensial. Kami mensimulasi pergerakan model dengan tiga negara dan menyatukan perbandingan analisa dinamis terhadap beberapa parameters. Analisa dinamika komparatif mendemonstrasikan bagaimana perekonomian global bereaksi untuk perubahan dalam utilisasi pengetahuan secara efisien, dampak dari return to scale dalam akumulasi modal, pengembalian pajak atas sektor barang modal, lintas batas polusi, pajak konsumsi, populasi penduduk, dan probabilitas menabung masa transisi maupun dalam jangka panjang.

Kata kunci: lintas batas polusi; pertumbuhan ekonomi global; perdagangan bebas; akumulasi kekayaan; dan akumulasi modal.

JEL Classification: F11, O44, F64

1. INTRODUCTION

This study deals with global dynamic interdependence between economic growth, economic structural changes, inequalities in income and wealth, knowledge creation, knowledge diffusion, knowledge utilization, environmental change with different tax policies. It builds a trade model with endogenous wealth, environment, knowledge accumulation and labor and capital distribution between sectors and between countries under perfectly competitive markets and free trade. The model is built on the basis of the Solow-Uzawa neoclassical growth model, the Oniki-Uzawa trade model, some neoclassical growth models with environment, ideas about transboundary pollution in environmental economics, Arrow's learning-by-doing model, and Zhang's idea about knowledge as international public stock. We integrate these approaches by applying the utility function proposed by Zhang (1993) to determine saving and consumption. As systematically explained by Zhang (2015), the approach avoids the Solow approach without microeconomic foundation (Solow, 1956) and the Ramsey approach by adding utility function over time. Zhang's approach makes it analytically easier (than the traditional Ramsey approach) to analyze behavior of households with endogenous saving and consumption.

The modelling of environmental dynamics is based on some growth models with environmental change. Dynamic relations between growth and environmental change have been formally analyzed since the publication of the semina papers by Ploude (1972) and Forster (1973). There are different economic factors that may affect environmental change (e.g., John and Pecchenino, 1994; Lamla, 2009; Prieur, 2009; Tsurumi and Managi, 2010; Gassebner et al. 2011; and Fairbrother, 2013). From the literature of environmental economics we know that environment is determined not only by behavior of firms, households, transboundary pollution and the government, but also affects behavior of firms, households, trade patterns and the government. Although economic growth may worsen environmental conditions, growth also implies a higher material standard of living which will, through the demand for a better environment induces changes in the structure of the economy to improve environment. As pointed out by Lin and Liscow (2012: 268): "The effect of increasing income on environmental quality is an issue that has long puzzled economists. For over decade, economists have theorized that a graph of environmental degradation versus income often looks something approximating an inverted-U shape, dubbed the environmental Kuznets curve (EKC) after Simon Kuznets' work in the 1950s and 1960s on income equality…." But a large number of empirical studies find different relations - for instance, inverted U-shaped relationship, a U-shaped relationship, a monotonically increasing or monotonically decreasing relationship - between pollution and rising per capita income levels (e.g., Bravo and Marelli, 2007). Another important aspect that has been often neglected in the literature of formal growth models with environment is related to international trade and transboundary pollution (e.g., Fullerton and Kim, 2008. This study deals with growth, trade, and environmental change within an integrated framework.

With regards to capital mobility and trade, our model is based on the neoclassical growth trade model. The national growth parts are based on the Solow growth model (Solow, 1956) and the Uzawa two-sector model (Uzawa, 1961, 1963). Early trade models with capital movements are originated by MacDougall (1960) and Kemp (1961). Those models in the 1960s are mostly static (e.g., Findlay, 1984). Most of trade models with endogenous capital and/or knowledge in the contemporary literature are either limited to two-country or small open economies (for instance, Grossman and Helpman, 1991; Wong, 1995; Jensen and Wong, 1998; Obstfeld and Rogoff, 1998). There are some growth models with international trade (e.g., Oniki and Uzawa, 1965; Bardhan, 1965; Deardorff and Hanson, 1978; Brecher et al., 2002; Nishimura and Shimomura, 2002; Ono and Shibata, 2005; and Chen et al., 2008). But none of these models with endogenous capital accumulation contain endogenous environmental changes and knowledge.

Our approach to technological change is based on Arrow's learning-by-doing. One of the first seminal attempts to render technical progress endogenous in growth models was made by Arrow (1962). He emphasizes one aspect of knowledge accumulation - learning by doing. There are different studies on endogenous human capital, technological change, and knowledge (e.g., Uzawa, 1965, Romer, 1986; Lucas, 1988; Aghion and Howitt, 1992; Manasse and Turrini, 2001; Agénor, 2004; Aghion et al. 2009; Gersbach et al. 2013). Nevertheless, each of these studies emphasizes some aspects of economic growth and neglects other some aspects. It is desirable to develop a comprehensive framework within which different aspects of knowledge can be examined in an integrated manner. This study takes account of various aspects of knowledge creation, diffusion and utilization in a single framework. We treat knowledge as an international public good in the sense that all countries access knowledge and the utilization of knowledge by one country does not affect that by others. The modelling of endogenous knowledge is based on Zhang (1992). Zhang's study showed the dynamic interdependence between capital accumulation, knowledge dynamics, and international trade. Nevertheless, it neglected endogenous environment and was concerned the analytical properties of the global economy with only two economies (without simulation). This paper develops a multi-country growth trade model with endogenous wealth, endogenous knowledge, and endogenous environment for a global economy with any number of national economies with free trade. The modeling of endogenous environment in this study is based on Zhang (2014). The rest of the paper is organized as follows. Section 2 defines the basic model by synthesizing Zhang's two models (1992, 2014). Section 3 shows how we solve the dynamics and simulates the motion of the global economy. Section 4 carries out comparative dynamic analysis to examine the impact of changes in some parameters on the motion of the global economy. Section 5 concludes the study. The appendix proves the main results in Section 3.

2. THE TRADE MODEL WITH WEALTH, KNOWLEDGE, AND ENVIRONMENT

The world economy consists of multiple countries, indexed by $j = 1, ..., J$. Country j has a fixed population, N_j, ($j = 1, ..., J$). Each country has three sectors: one capital good sector, one consumer good sector and one environmental sector. Following Zhang (2014), we assume that the national government financially supports the environmental sector. We follow the neoclassical growth theory to describe the production activities (e.g., Burmeister and Dobell, 1970; Azariadis, 1993; Barro and Sala-i-Martin, 1995). In particular, the capital good and consumer good sectors are the same as in the Uzawa two sector model (Uzawa, 1961). We further assume that all the countries produce the internationally homogenous capital good. This assumption follows the traditional Oniki-Uzawa trade model with endogenous capital and free trade. As reviewed by Ikeda and Ono (1992), most of trade models with endogenous capital are structured like Oniki-Uzawa trade model and its various extensions with one capital good. In our model, each country also has one consumer goods (and service) sector. The output of the consumer good sector is not tradable. The consumer good supplied by a country can be consumed only by the domestic consumers.

Households own assets and distribute their incomes to consume and save. The production sectors use capital and labor. Exchanges take place in perfectly competitive markets. Production sectors pay environmental taxes and sell their product to households or to other sectors. Households sell their labor and assets to production sectors. Factors are inelastically supplied and the available factors are fully utilized at every moment. Saving is undertaken only by households. This implies that all earnings of firms are distributed in the form of payments to factors of production. We omit the possibility of hoarding of output in the form of non-productive inventories held by households. We require savings and investment to be equal at any point in time.

Let prices be measured in terms of the capital good and the price of the capital good be unit. We denote wage and interest rates by $w_j(t)$ and $r_j(t)$, respectively, in the jth country. In the free trade system, the interest rate is identical throughout the world economy, i.e., $r(t) = r_j(t)$. The capital good is used as inputs in the three sectors. Capital depreciates at a constant exponential rate δ_j, being independent of the manner of use within each country. Let $p_j(t)$ denote the price of consumer good in country j. We use subscript index, i, s and e to stand for the capital good sector, the consumer good sector, and the environmental sector, respectively. We use $N_{jm}(t)$ and $K_{jm}(t)$ to stand for the labor force and capital stocks employed by sector m in country j. Let $F_{jm}(t)$ stand for the output level of sector m in country j.

The capital good sector

We assume that production of capital good is dependent on the labor force, physical capital, knowledge inputs. Following Zhang (1992), we assume that knowledge is an international public good. Knowledge can be applied by any sector without affecting any other sector. Knowledge is public good which can be freely used by different producers. We use the conventional production function to describe a relationship between inputs and output, except that knowledge and environment affects the total productivity. The production functions are specified as follows

$$F_{ji}(t) = A_{ji}\,\Gamma_{ji}(E_j)Z^{m_{ji}}(t)K_{ji}^{\alpha_{ji}}(t)N_{ji}^{\beta_{ji}}(t), \quad A_{ji},\ \alpha_{ji},\ \beta_{ji} > 0, \quad \alpha_{ji} + \beta_{ji} = 1, \tag{1}$$

where A_{ji}, α_{ji}, and β_{ji} are positive parameters. Here, $\Gamma_{ji}(E_j)$ is a function of the environmental quality measured by the level of pollution, $E_j(t)$, in country j. It is reasonable to assume that the productivity of the capital good sector is non-positively related to the pollution level, i.e., $d\,\Gamma_{ji} / d\,E_j \le 0$. In (1) $Z(t)\,(> 0)$ is the knowledge stock at time t. Here, we call m_{jq} (j, q)'s knowledge utilization efficiency parameter. If we interpret $Z^{m_{jq}/\beta_{jq}}(t)N_j(t)$ as (j, q)'s qualified labor force, we see that the production function is a neoclassical one and homogeneous of degree one with the inputs. We use τ_{ji} to stand for the fixed tax rate on the capital good sector. The marginal conditions of the capital good sector are given by

$$r(t) + \delta_k = \frac{\alpha_{ji}\,\bar{\tau}_{ji}\,F_{ji}(t)}{K_{ji}(t)}, \quad w_j(t) = \frac{\beta_{ji}\,\bar{\tau}_{ji}\,F_{ji}(t)}{N_{ji}(t)}, \tag{2}$$

where $\bar{\tau}_{ji} \equiv 1 - \tau_{ji},\ 0 < \tau_{ji} < 1$.

The consumer good sectors

The production functions of the consumer good sectors are

$$F_{js}(t) = A_{js}\,\Gamma_{js}(E_j(t))Z^{m_{js}}(t)K_{js}^{\alpha_{js}}(t)N_{js}^{\beta_{js}}(t), \quad \alpha_{js} + \beta_{js} = 1, \quad \alpha_{js},\ \beta_{js} > 0, \tag{3}$$

where A_{js}, α_{js}, and β_{js} are the technological parameters of the consumer good sector and $\Gamma_{ji}(E_j(t))$ is a function of the environmental quality. We use τ_{js} to stand for the fixed tax rate on the consumer good sector and introduce $\bar{\tau}_{js} \equiv 1 - \tau_{js},\ 0 < \tau_{js} < 1$. The marginal conditions are

$$r(t) + \delta_k = \frac{\alpha_{js}\,\bar{\tau}_{js}\,p_j(t)F_{js}(t)}{K_{js}(t)}, \quad w_j(t) = \frac{\beta_{js}\,\bar{\tau}_{js}\,p_j(t)F_{js}(t)}{N_{js}(t)}. \tag{4}$$

Environmental change

We now describe the dynamics of $E_j(t)$. Following Zhang (2014), we specify the dynamics of the stock of pollutants as follows

$$\dot{E}_j(t) = \theta_{ji} F_{ji}(t) + \theta_{js} F_{js}(t) + \theta_j C_j(t) - F_{je}(t) - \bar{\theta}_j E_j(t) + \Omega_j((E_q(t))), \tag{5}$$

in which θ_{ji}, θ_{jk}, θ_j, and $\bar{\theta}_j$ are positive parameters and

$$F_{je}(t) = A_{je} \Gamma_{je}(E_j(t)) K_{je}^{\tilde{\alpha}_{je}}(t) N_e^{\tilde{\beta}_{je}}(t), \quad A_{je}, \tilde{\alpha}_{je}, \tilde{\beta}_{je} > 0, \tag{6}$$

where A_{je}, α_{je}, and β_{je} are positive parameters, and $\Gamma_{je}(E_j) (\geq 0)$ is a function of E_j. The term $\theta_{ji} F_j$ means that pollutants that are emitted during production processes are linearly positively proportional to the output level (for instance, Dinda, 2005). The parameter, θ_{ji}, means that in consuming one unit of the good the quantity θ_{ji} is left as waste. Hence, $\theta_{ji} F_j$ is the creation of pollutants by the capital good sector. Similarly, $\theta_{js} F_{js}$ is the creation of pollutants by the consumer good sector. The creation of pollutants by consumers is given $\theta_j C_j$. The parameter $\bar{\theta}_j$ is called the rate of natural purification. The term $\bar{\theta}_j E_j$ measures the rate that the nature purifies environment. The term $K_{je}^{\tilde{\alpha}_e} N_{je}^{\tilde{\beta}_e}$ in F_{je} means that the purification rate of environment is positively related to capital and labor inputs. The function Γ_{je} implies that the purification efficiency is dependent on the stock of pollutants. It is not easy to generally specify how the purification efficiency is related to the scale of pollutants. For simplicity, we require Γ_e to be positively related to the stock of pollutants. In an economy where environment is heavily polluted, the environment sector is productive in the sense that some efforts may bring about great results. On the other hand, efforts to improve environment may have little impact on the clean environment. We measure effects of transboundary pollution by $\Omega_j((E_q))$ where (??) is a function of pollutants of all the countries. The functions are possibly related to many factors, such as distances between countries and wind directions. We will specify these functions when simulating the model. It should be noted that in the literature of trade and environment there are many studies which explicitly take account of transboundary pollution (e.g., Copeland and Taylor, 1994, 1995, 2003; Ono, 1998; Chao and Yu, 1999; Naito, 2003; Selden and Song, 1995; Takarada, 2005; Schweinberger and Woodland, 2008; Munro, 2009; Qiu and Yu, 2009; Abe, *et al.* 2012; Suhardiman and Giordano, 2012; Deng and Xu, 2015).

Knowledge creation

This study follows Zhang (1992) in modelling utilization and creation of knowledge. In Zhang's model, knowledge growth is through Arrow's learning-by-doing (Arrow, 1962). The equation for knowledge growth is given as follows

$$\dot{Z}(t) = \sum_{j=1}^{J} \left(\frac{\tilde{\tau}_{ji} F_{ji}(t)}{Z^{\varepsilon_{ji}}(t)} + \frac{\tilde{\tau}_{js} F_{js}(t)}{Z^{\varepsilon_{js}}(t)} \right) - \delta_z Z(t), \qquad (7)$$

in which $\delta_z \ (\geq 0)$ is the depreciation rate of knowledge, and ε_{jq} and $\tilde{\tau}_{jq}$ are parameters. We require $\tilde{\tau}_{jq}$ non-negative. We interpret $\tilde{\tau}_{jq} F(t) / Z^{\varepsilon_{jq}}(t)$ as the contribution to knowledge accumulation through learning by doing by country $j's$ capital good sector. To see how learning by doing occurs, assume that knowledge is a function of country $j's$ total industrial output during some period

$$Z(t) = a_1 \left\{ \int_0^t F_j(\theta) d\theta \right\}^{a_2} + a_3$$

in which a_1, a_2 and a_3 are positive parameters. The above equation implies that the knowledge accumulation through learning by doing exhibits decreasing (increasing) returns to scale in the case of $a_2 < (>) \ 1$. We interpret a_1 and a_3 as the measurements of the efficiency of learning by doing by the sector. Taking the derivatives of the equation yields

$$\dot{Z}(t) = \frac{\tilde{\tau}_{jj} F_j(t)}{Z^{\varepsilon_{jj}}(t)}$$

in which $\tau_{jj} \equiv a_1 a_2$ and $\varepsilon_{jj} \equiv 1 - a_2$.

Consumer behaviors

This study models household behavior with the approach proposed by Zhang (1993). In addition to the environmental taxation on firms (outputs), we also take account of taxation on wealth income and wage income. Consumers make decisions on choice of consumption levels of goods as well as on how much to save. Let $\bar{k}_j(t)$ stand for per capita wealth of country j. We have $\bar{k}_j(t) = \bar{K}_j(t) / N_j$, where $\bar{K}_j(t)$ is the total wealth held by country j. We use τ_{jk} and τ_{jw} to respectively stand for the tax rates on the wealth income and wage income. Per capita current income from the interest payment $r(t)\bar{k}_j(t)$ and the wage payment $w_j(t)$ is

$$y_j(t) = \bar{\tau}_{jk} \ r(t) \bar{k}_j(t) + \bar{\tau}_{jw} \ w_j(t),$$

where $\bar{\tau}_{jk} \equiv 1 - \tau_{jk}$ and The per capita disposable income $\hat{y}_j(t)$ is the sum of the current disposable income and the value of wealth. That is

$$\hat{y}_j(t) = y_j(t) + \bar{k}_j(t). \qquad (8)$$

The disposable income is used for saving and consumption. At each point in time, a consumer would distribute the total available budget between saving $s_j(t)$ and consumption $c_j(t)$. The budget constraint is given by

$$\left(1+\tau_{jc}\right)p_j(t)c_j(t)+s_j(t)=\hat{y}_j(t), \qquad (9)$$

where τ_{jc} is the tax rate on consumption. In the literature of environmental economics there are different taxes on households as well as producers (e.g., Bovenberg and Smulders, 1995; Bovenberg *et al.*, 2008).

The consumers choose two variables, $s_j(t)$ and $c_j(t)$, to maximize utilities. We specify the utility function as follows

$$U_j(t)=\Gamma_j\left(E_j(t)\right)c_j^{\xi_{0j}}(t)s_j^{\lambda_{0j}}, \quad \xi_{0j}, \lambda_{0j}>0, \qquad (10)$$

where ξ_{0j} is the propensity to consume and λ_{0j} the propensity to own wealth. Like in Balcao (2001) and Nakada (2004), we consider that $\Gamma_j\left(E_j(t)\right)$ is a function negatively related to the environment. In our model environmental change is affected by production activities, consumption activities, transboundary pollution, natural purification and government environmental policies.

Maximizing $U_j(t)$ subject to (9) yields

$$p_j(t)c_j(t)=\xi_j\,\hat{y}_j(t), \quad s_j(t)=\lambda_j\,\hat{y}_j(t), \qquad (11)$$

where

$$\xi_j\equiv\frac{\rho_j\,\xi_{0j}}{1+\tau_{jc}}, \quad \lambda_j\equiv\rho_j\,\lambda_{0j}, \quad \rho_j\equiv\frac{1}{\xi_{0j}+\lambda_{0j}}.$$

We now find dynamics of capital accumulation. According to the definition of $s_j(t)$ the change in the household's wealth is given by

$$\dot{\bar{k}}_j(t)=s_j(t)-\bar{k}_j(t). \qquad (12)$$

The equation simply states that the change in wealth is equal to saving minus dissaving.

The capital and labor employed by the environment sector

We now determine how the government determines the number of labor force and the level of capital employed for improving environment. For simplicity, we assume that the government's income is used up only for the environmental purpose. The government's tax incomes consist of the tax incomes on the production sector, consumption, wage income and wealth income. The government's income is

$$Y_{je}(t)=\tau_{ji}\,F_{ji}(t)+\tau_{js}\,F_{js}(t)+\tau_{jc}\,c_j(t)N_j+\tau_{jw}\,w_j(t)N_j+\tau_{jk}\,r(t)\bar{k}_j(t)N_j. \qquad (13)$$

As there are only two input factors in the environmental sector, the government budget is

$$\left(r(t)+\delta_k\right)K_{je}(t)+w_j(t)N_{je}(t)=Y_{je}(t). \qquad (14)$$

We need an economic mechanism to analyze how the government distributes the tax income. We assume that the government will employ the labor force and capital stocks for purifying environment in such a way that the purification rate achieves its maximum under the given budget constraint. The government's optimal problem is given by

$$\max_{\{K_{je}(t),\,N_{je}(t)\}} F_e(t) \qquad \text{s.t.: (14).}$$

The optimal solution is given by

$$(r(t) + \delta_k) K_{je}(t) = \alpha_{je} Y_{je}(t),$$
$$w_j(t) N_{je}(t) = \beta_{je} Y_{je}(t), \tag{15}$$

where

$$\alpha_{je} \equiv \frac{\tilde{\alpha}_{je}}{\tilde{\alpha}_{je} + \tilde{\beta}_{je}}, \quad \beta_{je} \equiv \frac{\tilde{\beta}_{je}}{\tilde{\alpha}_{je} + \tilde{\beta}_{je}}.$$

Demand and supply balance in service markets

The demand and supply equilibrium for the consumer good sector is

$$c_j(t) N_j = F_{js}(t), \quad j = 1, \ldots, J. \tag{16}$$

Factor marketing clearing conditions

We use $K(t)$ to stand for the capital stocks of the world economy. The total capital stock employed by country j, $K_j(t)$, is allocated between the three sectors. It should be noted that $K_j(t)$ may not equal $\bar{K}_j(t)$ which is the wealth owned by country j. As full employment of labor and capital is assumed, we have

$$K_{ji}(t) + K_{js}(t) + K_{je}(t) = K_j(t), \quad N_{ji}(t) + N_{js}(t) + N_{je}(t) = N_j. \tag{17}$$

Wealth balance

The wealth owned by the global population is equal to the total global wealth. That is

$$K(t) = \sum_{j=1}^{J} K_j(t) = \sum_{j=1}^{J} \bar{k}_j(t) N_j. \tag{18}$$

Market clearing in capital markets

The world production is equal to the world net savings. That is

$$S(t) - K(t) + \sum_{j=1}^{J} \delta_{jk} K_j(t) = F(t), \tag{19}$$

where

$$S(t) \equiv \sum_{j=1}^{J} s_j(t) N_j, \quad F(t) \equiv \sum_{j=1}^{J} F_j(t).$$

The trade balances of the economies are given by

$$B_j(t) = \left(\bar{K}_j(t) - K_j(t)\right) r(t). \tag{20}$$

When $B_j(t)$ is positive (negative), we say that country j is in trade surplus (deficit). When $B_j(t)$ is zero, country $j's$ trade is in balance.

We completed the model. Irrespective of the obvious strict assumptions in our model, from a structural point of view the model is quite general in the sense that some well-known models in economics can be considered as its special cases. For instance, if the population is homogeneous and environment is constant, our model is structurally similar to the neoclassical growth model by Solow (1956) and Uzawa (1961). Our model is also structurally similar to the Oniki-Uzawa trade model (Oniki and Uzawa, 1965). The model is a synthesis of Zhang's trade growth model with knowledge (Zhang, 1992) and trade model with environment (Zhang, 2014). As mentioned before, our approach is also based on some growth models in the literature of environmental economics.

3 THE WORLD ECONOMIC DYNAMICS

The dynamic system consists of any (finite) number of national economies. The dynamic system is of high dimension. The following lemma shows that the dimension of the dynamical system is $2J+1$. We also provide a computational procedure for calculating all the variables at any point in time. First we introduce new variables

$$z_1(t) \equiv \frac{r(t)+\delta_{k1}}{w_1(t)}, \ \left\{\bar{k}_j(t)\right\} \equiv \left(\bar{k}_2(t), \ldots, \bar{k}_J(t)\right).$$

Lemma

The dynamics of the world economy is governed by the following $2J+1$ dimensional differential equations system with $z_1(t)$, $\left\{\bar{k}_j(t)\right\}$, $Z(t)$, and $\left(E_j(t)\right)$ as the variables

$$\dot{z}_1(t) = \Phi_1\left(z_1(t), \left(E_j(t)\right), \left\{\bar{k}_j(t)\right\}, Z(t)\right),$$
$$\dot{\bar{k}}_j(t) = \Phi_j\left(z_1(t), \left(E_j(t)\right), \left\{\bar{k}_j(t)\right\}, Z(t)\right), \quad j = 2, \ldots, J,$$
$$\dot{E}_j(t) = \Omega_j\left(z_1(t), \left(E_j(t)\right), \left\{\bar{k}_j(t)\right\}, Z(t)\right), \quad j = 1, 2, \ldots, J,$$
$$\dot{Z}(t) = \Omega\left(z_1(t), \left(E_j(t)\right), \left\{\bar{k}_j(t)\right\}, Z(t)\right), \tag{20}$$

in which $\Phi_j(t)$ and $\Omega_j(t)$, $\Omega(t)$ are unique functions of $z_1(t)$, $\left\{\bar{k}_j(t)\right\}$, $\left(E_j(t)\right)$ and $Z(t)$ defined in Appendix. For any given positive values of $z_1(t)$, $\left\{\bar{k}_j(t)\right\}$, $\left(E_j(t)\right)$ and $Z(t)$ at any point in time, the other variables are uniquely determined by the following procedure: $r(t)$ and $w_j(t)$ by (A2) $\rightarrow$ $p_j(t)$ by (A4) $\rightarrow$ $\bar{k}_1(t)$ by (A19) $\rightarrow$ $K_j(t)$ by (A17) $\rightarrow$ $N_{ji}(t)$ and $N_{je}(t)$ by (A11) $\rightarrow$ $N_{js}(t)$ by (A7) $\rightarrow$ $K_{je}(t)$, $K_{js}(t)$, and $K_{ji}(t)$ by (A1) $\rightarrow$ $\hat{y}_j(t)$ by (A5) $\rightarrow F_{ji}(t)$, $F_{js}(t)$ and $F_{je}(t)$ by the definitions $\rightarrow$ $c_j(t)$ and $s_j(t)$ by (10) $\rightarrow$ $Y_{je}(t) = w_j(t)N_{je}(t)/\beta_{je} \rightarrow K(t) = \sum_j \bar{k}_j(t)N_j \rightarrow \bar{K}_j(t) = \bar{k}_j(t)N_j$ $\rightarrow B_j(t) = \left(\bar{K}_j(t) - K_j(t)\right)r(t) \rightarrow U_j(t)$ by the definitions.

The lemma provides a computational procedure for illustrating the motion of the economic system with any number of countries. As it is difficult to interpret the analytical results, to study properties of the system we simulate the model for a 3-country global economy. We specify the functions dependent on environmental quality as follows

$$\Gamma_{jm}\left(E_j(t)\right) = E_j^{-b_{jm}}(t), \ \Gamma_j\left(E_j(t)\right) = E_j^{-b_j}(t), \quad j = 1, 2, 3, \quad m = i, s, e.$$

We require $b_{ji}, b_{js}, b_j \geq 0$ and $b_{je} \leq 0$. The transboundary pollution functions are specified as

$$\Omega_q\left(\left(E_q(t)\right)\right) = \sum_{j,\, j \neq q}^{J} \theta_{jq}\, E_j(t).$$

It is reasonable to require $\theta_{jq} \geq 0$. The transboundary pollution functions imply that a country may be polluted by other countries and the speed is linearly related to the pollutant levels of these countries. We specify the parameters as follows:

$$\begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 10 \\ 30 \end{pmatrix}, \begin{pmatrix} m_{1i} \\ m_{2i} \\ m_{3i} \end{pmatrix} = \begin{pmatrix} 0.4 \\ 0.2 \\ 0.1 \end{pmatrix}, \begin{pmatrix} m_{1s} \\ m_{2s} \\ m_{3s} \end{pmatrix} = \begin{pmatrix} 0.4 \\ 0.2 \\ 0.1 \end{pmatrix}, \begin{pmatrix} A_{1i} \\ A_{2i} \\ A_{3i} \end{pmatrix} = \begin{pmatrix} 1.7 \\ 1 \\ 0.8 \end{pmatrix}, \begin{pmatrix} A_{1s} \\ A_{2s} \\ A_{3s} \end{pmatrix} = \begin{pmatrix} 1.5 \\ 0.9 \\ 0.7 \end{pmatrix}, \begin{pmatrix} A_{1e} \\ A_{2e} \\ A_{3e} \end{pmatrix} = \begin{pmatrix} 1.2 \\ 1 \\ 0.9 \end{pmatrix},$$

$$\begin{pmatrix} \alpha_{ji} \\ \alpha_{js} \\ \delta_z \end{pmatrix} = \begin{pmatrix} 0.31 \\ 0.33 \\ 0.05 \end{pmatrix}, \begin{pmatrix} \tilde{\tau}_1 \\ \tilde{\tau}_2 \\ \tilde{\tau}_3 \end{pmatrix} = \begin{pmatrix} 0.03 \\ 0.02 \\ 0.01 \end{pmatrix}, \begin{pmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \\ \varepsilon_{3i} \end{pmatrix} = \begin{pmatrix} 0.02 \\ 0.4 \\ 0.4 \end{pmatrix}, \begin{pmatrix} b_{1i} \\ b_{2i} \\ b_{3i} \end{pmatrix} = \begin{pmatrix} b_{1s} \\ b_{2s} \\ b_{3s} \end{pmatrix} = \begin{pmatrix} b_{1e} \\ b_{2e} \\ b_{3e} \end{pmatrix} = \begin{pmatrix} 0.1 \\ 0.02 \\ 0.01 \end{pmatrix},$$

$$\begin{pmatrix} \lambda_{10} \\ \lambda_{20} \\ \lambda_{30} \end{pmatrix} = \begin{pmatrix} 0.6 \\ 0.55 \\ 0.5 \end{pmatrix}, \begin{pmatrix} \bar{\theta}_1 \\ \bar{\theta}_2 \\ \bar{\theta}_3 \end{pmatrix} = \begin{pmatrix} 0.08 \\ 0.12 \\ 0.11 \end{pmatrix}, \begin{pmatrix} \tau_{1k} \\ \tau_{2k} \\ \tau_{3k} \end{pmatrix} = \begin{pmatrix} \tau_{1w} \\ \tau_{2k} \\ \tau_{3k} \end{pmatrix} = \begin{pmatrix} 0.01 \\ 0.02 \\ 0.02 \end{pmatrix}, \begin{pmatrix} \xi_{0j} \\ \tau_{jc} \\ \tau_{ji} \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.01 \\ 0.01 \end{pmatrix}, \begin{pmatrix} \tau_{js} \\ \tilde{\alpha}_{je} \\ \tilde{\beta}_{je} \end{pmatrix} = \begin{pmatrix} 0.01 \\ 0.4 \\ 0.2 \end{pmatrix},$$

$$\begin{pmatrix} b_j \\ \theta_{ji} \\ \theta_j \end{pmatrix} = \begin{pmatrix} 0.01 \\ 0.08 \\ 0.03 \end{pmatrix}, \begin{pmatrix} b_j \\ \theta_{js} \\ \theta_j \end{pmatrix} = \begin{pmatrix} 0.01 \\ 0.1 \\ 0.03 \end{pmatrix}, \begin{pmatrix} \delta_{1k} \\ \delta_{2k} \\ \delta_{3k} \end{pmatrix} = \begin{pmatrix} 0.05 \\ 0.04 \\ 0.04 \end{pmatrix}, \theta_{jq} = 0.01, \ \ j \neq q, \ \ j, q = 1, 2, 3. \qquad (21)$$

Country 1, 2 and 3's populations are respectively 3, 10, and 30. Country 3 has the largest population. Country 1, 2 and 3's total factor productivities of the capital good sector, A_{ji}, are respectively 1.7, 1, and 0.8. Country 1, 2 and 3's total productivities of the consumer good sectors, A_{js}, are respectively 1.5, 0.9, and 0.7. Country 1, 2 and 3's total productivities of the environmental sector, A_{je}, are respectively 1.2, 1, and 0.9. Country 1 has highest total productivity; country 2 next and country 3 the lowest. We call the three countries respectively as developed, industrializing, and underdeveloped economies (DE, IE, UE). We specify the values of the parameters, α_{ji}, in the Cobb-Douglas productions approximately 0.3. The DE's propensity to save is highest; country 2 next and country 3 the lowest. We require the tax rates on consumption level of any country to be one percent. The tax rates on other sectors and wealth are one or two percent. We plot the motion of the system under (17) with the following initial conditions

$$Z(0) = 1.2, \ \ z_1(0) = 0.1, \ \ \bar{k}_2(0) = 3.7, \ \ \bar{k}_3(0) = 2.2, \ \ E_1(0) = 11.5, \ \ E_2(0) = 11.3, \ \ E_3(0) = 60.$$

The motion of the variables is plotted in Figure 1. In Figure 1, the global output is

$$Y(t) = \sum_j \{F_{ji}(t) + p_j(t) F_{js}(t)\}.$$

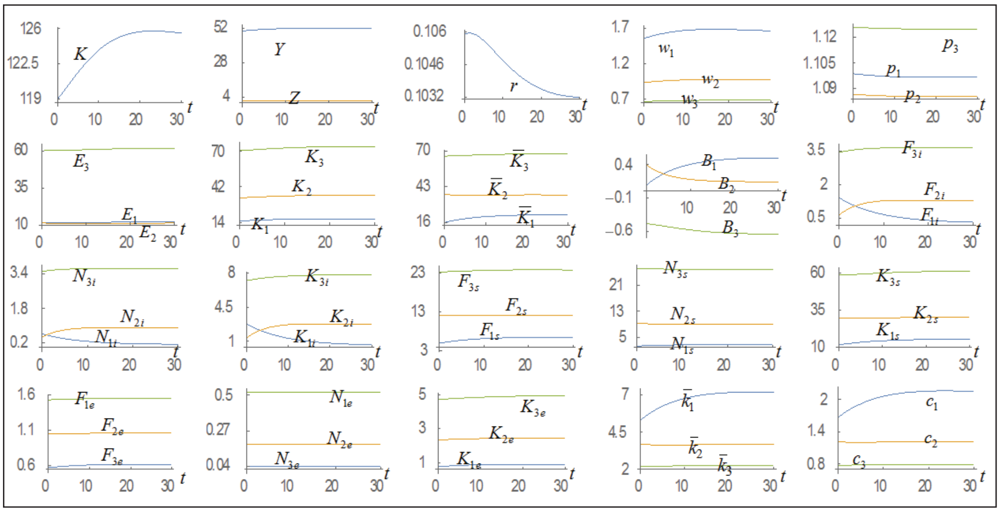

Figure 1. The Motion of the Economic System

From Figure 1 we see that the global output and capital stocks are increased over time. The knowledge is slightly changed. The different sectors in different countries experience different paths of economic development. The output levels of the capital good sectors in the DE and IE are increased. The output level of the capital good sector in the UE is reduced. The output levels of the consumer good sectors in the DE and UE are increased. The output level of the consumer good sector in the IE is reduced. The output levels of the environment sectors in the three economies are increased. The rate of interest falls and the wage rates in all the three economies are slightly change. The IE's and UE's trade balances are deteriorated. The DE's trade balance is improved. The prices of services in the three economies fall. The environmental conditions are slightly changed in the three economies.

The figure also demonstrates that the DE's per capita wealth and consumption level rise over time. The other two economies' per capita wealth and consumption levels are almost constant. This implies that over time the income and wealth gaps between the rich and the poor in terms of per household are enlarged. It should be noted that much of the discussion of income convergence in the literature of economic growth and development is based on the insights from analyzing models of closed economies (Barro and Sala-i-Martin, 1995). It is almost meaningless to discuss issues related to global income and wealth convergence from analytical results derived from frameworks without international interactions. Nevertheless, there are few growth models with endogenous wealth and trade on the basis of microeconomic foundation. For instance, the Solow model of closed economies predicts that convergence in income levels among closed countries is achieved by faster accumulation of physical capital in the poor countries. Nevertheless, if poor countries are opened to trade, the convergence may be stopped. From our result we also see that also implies the necessity of introducing multiple countries into the dynamic analysis of environment change. For instance, in a study by Grossman and Krueger (1995), they pay attention to the relationship per capita and various environmental indicators - urban air pollution, the state of the oxygen regime, in river basins, and fecal contamination of river basins, and contamination of river basins by heavy metals. They claim that they find no evidence that "environmental quality deteriorates steadily with economic growth." Our simulation indicates that although the conclusion made by Crossman and Krueger holds for some national economies, but is invalid for some other economies.

We also simulated the model with other initial conditions. The simulation results show that all the variables tend to become stationary over time with different initial conditions. We identify the following equilibrium point

$$K = 124.5, \quad Z = 1.31, \quad Y = 51.8, \quad r = 0.103,$$

$$\begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix} = \begin{pmatrix} 12.1 \\ 11.2 \\ 61.5 \end{pmatrix}, \begin{pmatrix} B_1 \\ B_2 \\ B_3 \end{pmatrix} = \begin{pmatrix} 0.49 \\ 0.14 \\ -0.63 \end{pmatrix}, \begin{pmatrix} Y_{1e} \\ Y_{2e} \\ Y_{3e} \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.52 \\ 1.06 \end{pmatrix}, \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} 1.1 \\ 1.09 \\ 1.13 \end{pmatrix}, \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 1.64 \\ 0.97 \\ 0.68 \end{pmatrix},$$

$$\begin{pmatrix} F_{1i} \\ F_{2i} \\ F_{3i} \end{pmatrix} = \begin{pmatrix} 0.35 \\ 1.27 \\ 3.62 \end{pmatrix}, \begin{pmatrix} F_{1s} \\ F_{2s} \\ F_{3s} \end{pmatrix} = \begin{pmatrix} 6.35 \\ 12.02 \\ 23.64 \end{pmatrix}, \begin{pmatrix} F_{1e} \\ F_{2e} \\ F_{3e} \end{pmatrix} = \begin{pmatrix} 0.62 \\ 1.06 \\ 1.55 \end{pmatrix}, \begin{pmatrix} N_{1i} \\ N_{2i} \\ N_{3i} \end{pmatrix} = \begin{pmatrix} 0.14 \\ 0.9 \\ 3.62 \end{pmatrix}, \begin{pmatrix} N_{1s} \\ N_{2s} \\ N_{3s} \end{pmatrix} = \begin{pmatrix} 2.82 \\ 8.92 \\ 25.86 \end{pmatrix},$$

$$\begin{pmatrix} N_{1e} \\ N_{2e} \\ N_{3e} \end{pmatrix} = \begin{pmatrix} 0.04 \\ 0.18 \\ 0.52 \end{pmatrix}, \begin{pmatrix} K_{1i} \\ K_{2i} \\ K_{3i} \end{pmatrix} = \begin{pmatrix} 0.69 \\ 2.73 \\ 7.74 \end{pmatrix}, \begin{pmatrix} K_{1s} \\ K_{2s} \\ K_{3s} \end{pmatrix} = \begin{pmatrix} 14.8 \\ 29.7 \\ 60.6 \end{pmatrix}, \begin{pmatrix} K_{1e} \\ K_{2e} \\ K_{3e} \end{pmatrix} = \begin{pmatrix} 0.88 \\ 2.43 \\ 4.91 \end{pmatrix}, \begin{pmatrix} \bar{k}_1 \\ \bar{k}_2 \\ \bar{k}_3 \end{pmatrix} = \begin{pmatrix} 7.04 \\ 3.62 \\ 2.24 \end{pmatrix},$$

$$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 2.12 \\ 1.2 \\ 0.79 \end{pmatrix}.$$

It is straightforward to calculate the seven eigenvalues at the equilibrium point as follows

$$-0.207,\ -0.184,\ -0.132,\ -0.126,\ -0.076 \pm 0.07,\ -0.068.$$

The real parts of the eigenvalues are negative. The equilibrium point is stable. This conclusion is important as it guarantees that we can effectively carry out comparative dynamic analysis.

4. COMPARATIVE DYNAMIC ANALYSIS

We simulated the motion of the dynamic system. It is important to conduct comparative dynamic analysis. First, we introduce a symbol $\overline{\Delta} x_j(t)$ which stands for the change rate of the variable, $x_j(t)$, in percentage due to a given change in the parameter value.

The DE's capital good sector utilizing knowledge more efficiently

We now study the case when the environmental tax rate on the capital good sector is increased in the developed economy as follows: $m_{1i}: 0.4 \Rightarrow 0.42$. The simulation result is plotted in Figure 2. As the DE's capital good sector applies knowledge more efficiently, the sector initially augments the output level but reduces the output level in the long term. The capital good sectors of the other two economies initially reduce the output levels but enhance the output levels in the long term. The DE's consumer good sector initially lowers the output level but raises the output level in the long term. The consumer good sectors of the other two economies enhance the output levels in the long term. The global output and capital stock are augmented initially lowers but increased in the long term. The knowledge stock is increased. The rate of interest rises initially but falls in the long term. The DE's wage rate is increased. The other two economies' wage rates fall initially and rise in the long term. The DE's price of consumer good is increased and the other two economies' prices are slightly affected. In the long term all the economies have more wealth and use more capital inputs. The trade balances are slightly affected in the long term. The environment conditions are deteriorated in all the economies. The DE's per capita wealth and consumption level fall initially and rise in the long term. The other two economies' per capita wealth and consumption levels are slightly augmented. This implies that the income and wealth gaps between the rich and the poor in terms of per household are enlarged in the long term.

The UE's capital good sector raising the return to scale effect in knowledge accumulation

We now study what will happen to the global economy when the UE's capital good sector raises the return to scale effect in knowledge accumula-

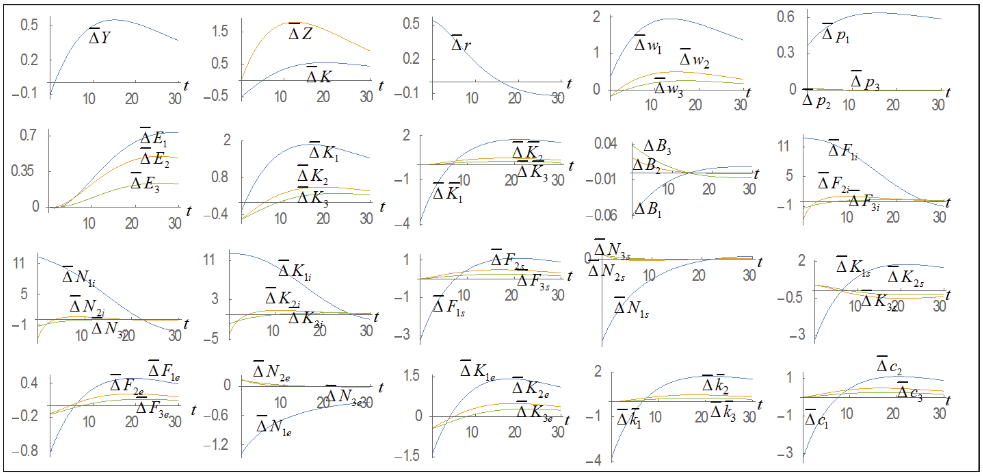

Figure 2. The DE's Capital Good Sector Utilizing Knowledge More Efficiently

tion as follows: $\varepsilon_{3i} : 0.4 \Rightarrow 0.3$. The simulation result is plotted in Figure 3. As the return to scale is strengthened in the UE's capital good sector's knowledge accumulation, the knowledge stock is increased. The global capital stock and output are increased. Both the wage rates and rate of interest are enhanced. The prices of consumer goods fall slightly. The trade balances are slightly affected. The capital stocks employed and owned by each economy are increased. The outputs of the three sectors in each economy are augmented. There are also redistributions of the labor force in each economy. The DE's per capita wealth and consumption level in each economy are increased.

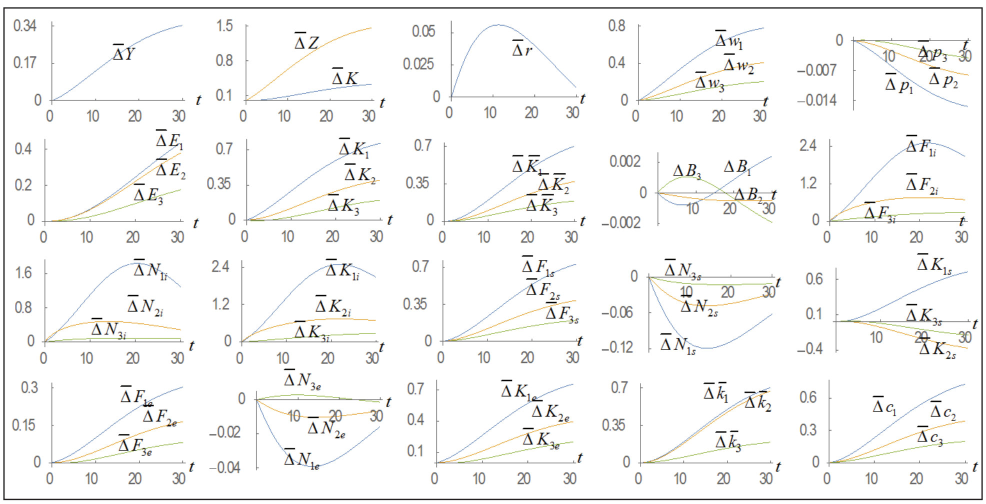

Figure 3. The Return to Scale Effect in Knowledge Accumulation Being Increased

The DE raising the tax rate on the capital good sector

We now examine what happens in the global economy if the tax rate on the capital good sector is increased in the DE as follows: $\tau_{1i} : 0.01 \Rightarrow 0.02$. The simulation result is plotted in Figure 4. The global capital and output levels are increased initially but reduced in the long term. The knowledge stock falls. The environment conditions in the three economies are improved. The change in the DE's tax rate changes the economic structure of the global economy. The output level of the DE's capital good sector is reduced in the short term and increased slightly in the long term. The output levels of the other two countries' capital good sectors are slightly changed in the long term. Per household's consumption level and wealth of the DE are reduced in the three economies.

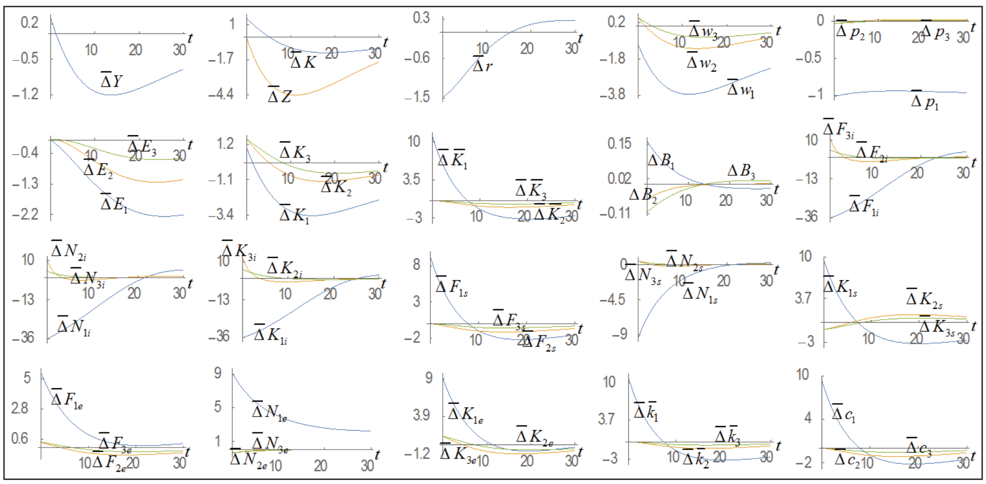

Figure 4. The DE Raising the Tax Rate on the Capital Good Sector

The transboundary pollution from the DE and IE to the UE becoming stronger

We now consider deal with the situation when both the DE and IE more strongly pollute the UE in the following way: $\theta_{13} : 0.01 \Rightarrow 0.05$ and $\theta_{23} : 0.01 \Rightarrow 0.05$. and The simulation result is plotted in Figure 5. As the other economies more strongly pollute the UE, the environment conditions are deteriorated in the three economies. The global capital, knowledge stock and global output fall. The capital good and consumer good sectors in the three economies reduce the output levels. Per household's consumption level and wealth of the DE are reduced in the three economies.

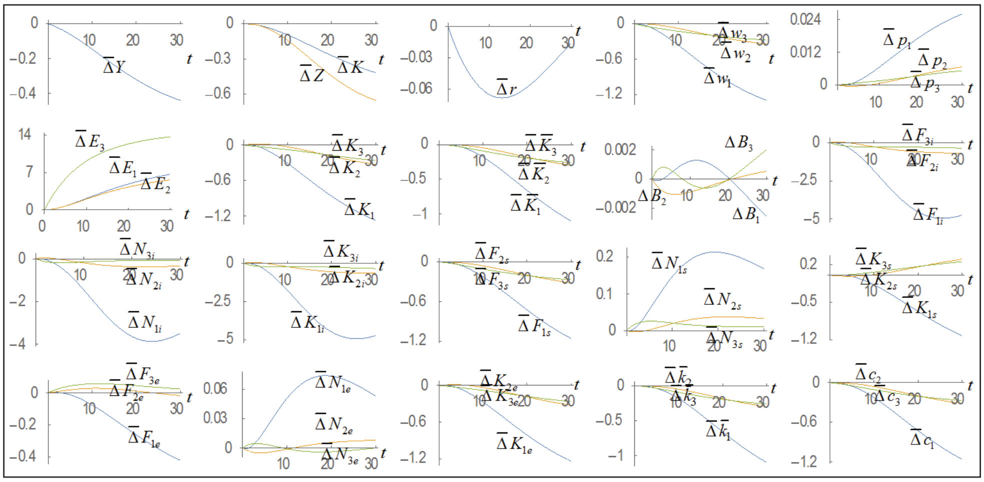

Figure 5. The Transboundary Pollution from the DE and IE to the UE Becoming Stronger

The tax rate on consumption being increased in the UE

We now study what happens in the global economy if the DE raises the consumption tax rate as follows: $\tau_{1c} : 0.01 \Rightarrow 0.04$. The simulation result is plotted in Figure 6. The environment conditions are deteriorated in the three economies. The global capital, knowledge stock and global output are increased. The rate of interest rises and the wage rates are increased. The prices of the consumer goods in the three economies are slightly reduced. Each household has more wealth. The households in the IE and UE consume more. The household in the DE consumes initially and more in the long term.

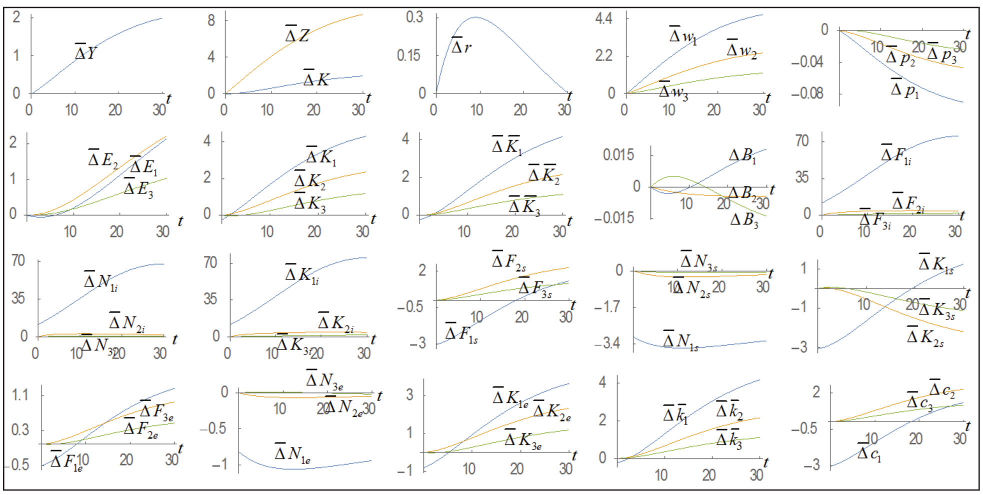

Figure 6. The Consumption Tax Rate Being Increased in the DE

A rise in the UE's population

It has been observed that the effect of population growth varies with the level of economic development (e.g., Galor and Weil, 1999; Boucekkine, et al., 2002). We now consider the case that the DE's population is increased as follows: $N_3 : 30 \Rightarrow 32$. The simulation result plotted in Figure 7 shows that as the population increases, the environmental qualities deteriorate in all the three phases of economies. The wage rates in the three phases of economies reduce initially but show increasing trend in the long term. The global wealth, knowledge stock and global output are augmented. The DE's and IE's trade balances are improved. The DE's trade balance is deteriorated. The prices are slightly affected. The households' wealth and consumption levels are increased in the three economies. The positive consequences of the population increase are due to the introduction of endogenous knowledge. The effects on the other variables are plotted in Figure 7.

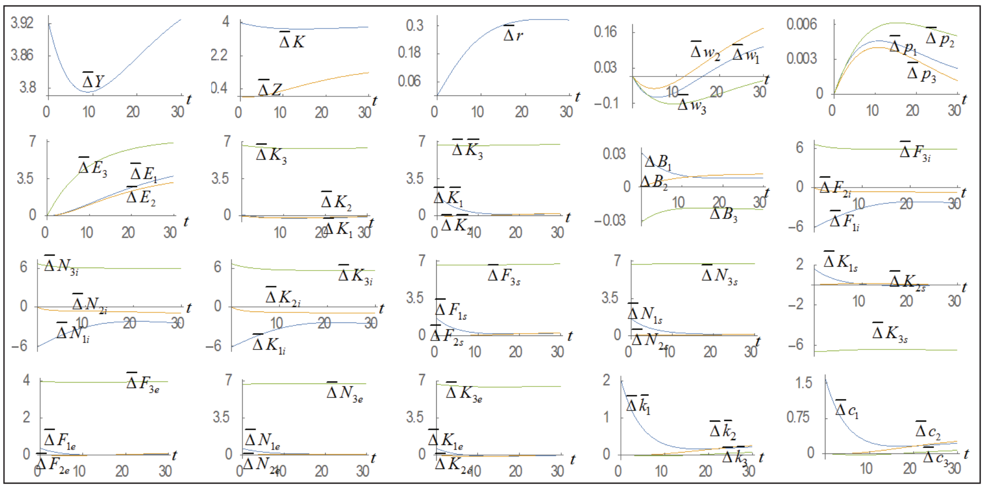

Figure 7. A Rise in the UE's Population

The DE household's propensity to save being increased

We now examine the effects of the following rise in the DE household's propensity to save on the global economy: $\lambda_{01} : 0.6 \Rightarrow 0.61$. The simulation result is plotted in Figure 8. Each household has more wealth. The households in the IE and UE consume more. The household in the DE consumes initially and more in the long term. The global income and capital stock rise. The knowledge stock falls. The rate of interest falls and the wage rates rise. The DE's trade balance is slightly improved and the IE's and UE's trade balances are deteriorated. The capital stocks employed and owned by each country are increased in the long term. The environment conditions are deteriorated in the long term. The output levels of the IE's and UE's two sectors are increased. The DE's output level of the capital good sector rises initially and falls in the long term. The DE's output level of the consumer good sector falls initially and rises in the long term.

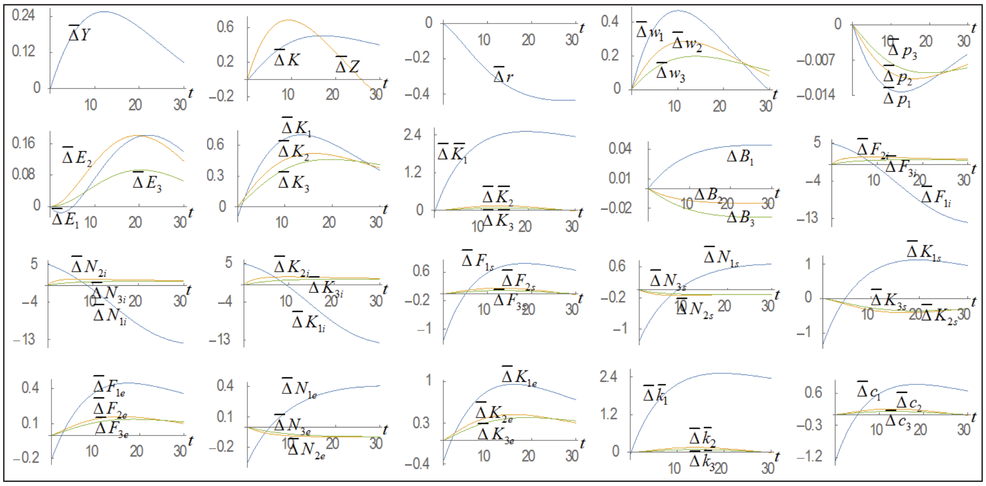

Figure 8. The DE Household's Propensity to Save Being Increased

5. CONCLUSIONS

The paper built a trade model with endogenous wealth, environment, knowledge accumulation and labor and capital distribution between sectors and between countries under perfectly competitive markets and free trade. The model is built on the basis of the Solow-Uzawa neoclassical growth model, the Oniki-Uzawa trade model, some neoclassical growth models with environment, ideas about transboundary pollution in environmental economics, Arrow's learning-by-doing model, and Zhang's idea about knowledge as international public stock. We integrated these approaches by applying the utility function proposed by Zhang (1993) to determine saving and consumption. The dynamics of J -country world economy is controlled by $2J+1$ differential equations. We simulated the motion of the model with three countries. We carried out comparative dynamic analysis with regard to changes in knowledge utilization efficiently, the return to scale effect in knowledge accumulation, the tax rate on the capital good sector, the transboundary pollution, the tax rate on consumption, the population, and the propensity to save. We described the dynamics of the economic system in the transitory processes as well as in the long term.

This study could have comprehensively discussed some important issues related to interdependence between capital accumulation, knowledge growth and environment change because we provided the computational procedure to follow the motion of the global economy. The model in this study is built with many strict assumptions. The model in this study may be generalized in different ways. We can study behavior of the dynamic system when the utility functions or/ and production functions are taken on other forms. The Solow model and Uzawa two-sector growth models are the two key models in the neoclassical economic growth theory and the Oniki-Uzawa growth model is a main key model of global economic dynamics with capital accumulation. These models have been generalized and extended in different ways. This study does not consider public goods and services. Learning by doing is taken into account only in terms of knowledge accumulation. There are other sources of knowledge growth, such as research and education, which should be taken into account.

Appendix: Proving Lemma 1

By (2), (4) and (15), we obtain

$$z_j \equiv \frac{r + \delta_k}{w_j} = \frac{N_{jm}}{\bar{\beta}_{jm} K_{jm}}, \quad j = 1, \ldots, J, \quad m = i, s, e, \qquad \text{(A1)}$$

where $\bar{\beta}_{jm} \equiv \beta_{jm} / \alpha_{jm}$. Insert (A1) in (2)

$$r = \alpha_{jr} \Gamma_{ji} Z^{m_{ji}} z_j^{\beta_{ji}} - \delta_k, \quad w_j = \alpha_j \Gamma_{ji} Z^{m_{ji}} z_j^{-\alpha_{ji}}, \qquad \text{(A2)}$$

where

$$\alpha_{jr} = \alpha_{ji} \bar{\tau}_{ji} \bar{\beta}_{ji}^{\beta_{ji}} A_{ji}, \quad \alpha_j = \frac{\beta_{ji} \bar{\tau}_{ji} A_{ji}}{\bar{\beta}_{ji}^{\alpha_{ji}}}.$$

From (A2) we also have

$$r = \alpha_{jr} \Gamma_{ji} Z^{m_{ji}} z_j^{\beta_{ji}} - \delta_{jk} = \alpha_{1r} \Gamma_{1i} Z^{m_{1i}} z_1^{\beta_{1i}} - \delta_{1k}, \quad j = 1, \ldots., J.$$

From the above equations we solve

$$z_j\left(z_1, \left(E_j\right)\right) = \left(\frac{\alpha_{1r} \Gamma_{1i} Z^{m_{1i}} z_1^{\beta_{1i}} + \delta_{jk} - \delta_{1k}}{\alpha_{jr} Z^{m_{ji}} \Gamma_{ji}}\right)^{1/\beta_{ji}}, \quad j = 2, \ldots., J. \qquad \text{(A3)}$$

Hence, we determine r, w_j, and z_j, as functions of z_1 and $\left(E_j\right)$. From (3) and (4), we have

$$p_j\left(z_1, \left(E_j\right)\right) = \frac{\bar{\beta}_{js}^{\alpha_{js}} z_j^{\alpha_{js}} w_j}{\beta_{js} \bar{\tau}_{js} A_{js} \Gamma_{js} Z^{m_{js}}}. \qquad \text{(A4)}$$

From (A4) and the definitions of $\hat{y}_j$, we have

$$\hat{y}_j = \left(1 + \bar{\tau}_{jk} r\right) \bar{k}_j + \bar{\tau}_{jw} w_j. \qquad \text{(A5)}$$

Insert $p_j c_j = \xi_j \hat{y}_j$ in (16)

$$\xi_j N_j \hat{y}_j = p_j F_{js}. \qquad \text{(A6)}$$

Substituting (A5) in (A6) yields

$$N_{js} = g_j \bar{k}_j + \bar{g}_j, \qquad \text{(A7)}$$

where we use $w_j N_{js} = \beta_{js} \bar{\tau}_{js} p_j F_{js}$ and

$$g_j\left(z_j, Z, (E_j)\right) \equiv \left(\frac{1+\bar{\tau}_{jk} r}{w_j}\right)\xi_j \beta_{js} \bar{\tau}_{js} N_j, \quad \bar{g}_j \equiv \bar{\tau}_{jw} \xi_j \beta_{js} \bar{\tau}_{js} N_j .$$

From (A1) and (17), we get

$$\frac{N_{ji}}{\bar{\beta}_{ji}} + \frac{N_{js}}{\bar{\beta}_{js}} + \frac{N_{je}}{\bar{\beta}_{je}} = z_j K_j . \qquad \text{(A8)}$$

Insert (A7) in (A8)

$$\frac{N_{ji}}{\bar{\beta}_{ji}} + \frac{N_{je}}{\bar{\beta}_{je}} = z_j K_j - \frac{g_j \bar{k}_j}{\bar{\beta}_{js}} - \frac{\bar{g}_j}{\bar{\beta}_{js}}. \qquad \text{(A9)}$$

Insert (A7) in $N_{ji} + N_{js} + N_{je} = N_j$

$$N_{ji} + N_{je} = N_j - g_j \bar{k}_j - \bar{g}_j . \qquad \text{(A10)}$$

Solve (A9) and (A10) with N_{ji} and N_{js} as the variables

$$N_{ji} = a_{ji} + \tilde{b}_{ji} \bar{k}_j - \bar{\beta}_j z_j K_j , \qquad \text{(A11)}$$
$$N_{je} = a_{je} + \tilde{b}_{je} \bar{k}_j + \bar{\beta}_j z_j K_j ,$$

where

$$a_{ji} \equiv \left(\frac{N_j - \bar{g}_j}{\bar{\beta}_{je}} + \frac{\bar{g}_j}{\bar{\beta}_{js}}\right)\bar{\beta}_j, \quad \tilde{b}_{ji}\left(z, (E_j)\right) \equiv \left(\frac{1}{\bar{\beta}_{js}} - \frac{1}{\bar{\beta}_{je}}\right)\bar{\beta}_j g_j ,$$

$$a_{je} \equiv -\left(\frac{\bar{g}_j}{\bar{\beta}_{js}} + \frac{N_j - \bar{g}_j}{\bar{\beta}_{ji}}\right)\bar{\beta}_j, \quad \tilde{b}_{je}\left(z, (E_j)\right) \equiv \left(\frac{1}{\bar{\beta}_{ji}} - \frac{1}{\bar{\beta}_{js}}\right)\bar{\beta}_j g_j , \quad \bar{\beta}_j \equiv \left(\frac{1}{\bar{\beta}_{je}} - \frac{1}{\bar{\beta}_{ji}}\right)^{-1}.$$

Substituting (A1) into (2) and (5) yields

$$F_{ji} = \frac{A_{ji} \Gamma_{ji} Z^{m_{ji}} N_{ji}}{\bar{\beta}_{ji}^{\alpha_{ji}} z_j^{\alpha_{ji}}}, \quad F_{js} = \frac{A_{js} \Gamma_{js} Z^{m_{js}} N_{js}}{\bar{\beta}_s^{\alpha_s} z_j^{\alpha_{js}}}. \qquad \text{(A12)}$$

Insert (A12) in (12)

$$Y_{je} = \Lambda_{ji} N_{ji} + \Lambda_{js} N_{js} + \tau_{je} c_j N_j + \tau_{jw} w_j N_j + \tau_{jk} r \bar{k}_j N_j . \qquad \text{(A13)}$$

where

$$\Lambda_{ji}\left(z_1, (E_j)\right) \equiv \frac{\tau_{ji} A_{ji} Z^{m_{ji}} \Gamma_{ji}}{\bar{\beta}_{ji}^{\alpha_{ji}} z_j^{\alpha_{ji}}}, \quad \Lambda_{js}\left(z_1, (E_j)\right) \equiv \frac{\tau_{js} A_{js} Z^{m_{js}} \Gamma_{js}}{\bar{\beta}_{js}^{\alpha_{js}} z_j^{\alpha_{js}}}.$$

From $p_j c_j = \xi_j \hat{y}_j$ and (A5), we have

$$c_j = \left(\frac{1+\bar{\tau}_{jk} r}{p_j}\right)\xi_j \bar{k}_j + \frac{\bar{\tau}_{jw} \xi_j w_j}{p_j}. \qquad \text{(A14)}$$

Substituting (A14) into (A13) yields

$$Y_{je} = \bar{\Lambda}_j + \Lambda_{ji} N_{ji} + \Lambda_{js} N_{js} + \Lambda_j \bar{k}_j , \tag{A15}$$

where

$$\Lambda_j\left(z_1, \left(E_j\right)\right) \equiv \left(\frac{1 + \bar{\tau}_{jk} r}{p_j}\right) \xi_j \tau_{jc} N_j + \tau_{jk} r N_j , \quad \bar{\Lambda}_j\left(z_1, \left(E_j\right)\right) \equiv \left(\frac{\bar{\tau}_{jw} \xi_j \tau_{jc}}{p_j} + \tau_{jw}\right) w_j N_j .$$

Insert (A15) in $w_j N_{je} = \beta_{je} Y_{je}$

$$\frac{w_j N_{je}}{\beta_{je}} = \bar{\Lambda}_j + \Lambda_{ji} N_{ji} + \Lambda_{js} N_{js} + \Lambda_j \bar{k}_j . \tag{A16}$$

Substituting (A7) and (A11) into (A16) yields

$$K_j = \bar{\Delta}_j + \Delta_j \bar{k}_j , \tag{A17}$$

where

$$\bar{\Delta}_j\left(z_1, \left(E_j\right)\right) \equiv \left(\bar{\Lambda}_j + \Lambda_{js} \bar{g}_j - \frac{w_j a_{je}}{\beta_{je}} + a_{ji} \Lambda_{ji}\right)\left(\frac{w_j}{\beta_{je}} + \Lambda_{ji}\right)^{-1} \frac{1}{\bar{\beta}_j z_j},$$

$$\Delta_j\left(z_1, \left(E_j\right)\right) \equiv \left(\tilde{b}_{ji} \Lambda_{ji} + g_j \Lambda_{js} + \Lambda_j - \frac{w_j \tilde{b}_{je}}{\beta_{je}}\right)\left(\frac{w_j}{\beta_{je}} + \Lambda_{ji}\right)^{-1} \frac{1}{\bar{\beta}_j z_j}.$$

Insert (A17) in (18)

$$\sum_{j=1}^{J} \bar{\Delta}_j + \sum_{j=1}^{J} \Delta_j \bar{k}_j = \sum_{j=1}^{J} \bar{k}_j N_j . \tag{A18}$$

Solve (A18) with $\bar{k}_1$ as the variable

$$\bar{k}_1 = \varphi\left(z_1, \left(E_j\right), \left\{\bar{k}_j\right\}\right) \equiv \left(\sum_{j=1}^{J} \bar{\Delta}_j + \sum_{j=2}^{J} \left(\Delta_j - N_j\right) \bar{k}_j\right) \frac{1}{N_1 - \Delta_1}. \tag{A19}$$

It is straightforward to confirm that all the variables can be expressed as functions of $\left(E_j\right)$, and $\left\{\bar{k}_j\right\}$ by the following procedure: r and w_j by (A2) $\rightarrow$ p_j by (A4) $\rightarrow$ $\bar{k}_1$ by (A19) $\rightarrow$ K_j by (A17) $\rightarrow$ N_{ji} and N_{je} by (A11) $\rightarrow$ N_{js} by (A7) $\rightarrow$ K_{je}, K_{js}, and K_{ji} by (A1) $\rightarrow$ $\hat{y}_j$ by (A5) $\rightarrow F_{ji}$, F_{js} and F_{je} by the definitions $\rightarrow$ c_j and s_j by (11) $\rightarrow$ $Y_{je} = w_j N_{je} / \beta_{je}$ $\rightarrow K = \sum_j \bar{k}_j N_j \rightarrow \bar{K}_j = \bar{k}_j N_j$ $\rightarrow$ $B_j = \left(\bar{K}_j - K_j\right) r$ $\rightarrow U_j$ by the definitions. From this procedure, (A19), (5), (7) and (12), we have

$$\dot{\bar{k}}_1 = \overline{\Phi}_1\left(z_1, \left(E_j\right), Z, \left\{\bar{k}_j\right\}\right) \equiv \lambda_1 \hat{y}_1 - \varphi, \tag{A20}$$

$$\dot{\bar{k}}_j = \Phi_j\left(z_1, \left(E_j\right), Z, \left\{\bar{k}_j\right\}\right) \equiv \lambda_j \hat{y}_j - \bar{k}_j, \quad j = 2, \ldots, J,$$

$$\dot{Z} = \Omega\left(z_1, \left(E_j\right), Z, \left\{\bar{k}_j\right\}\right) \equiv \sum_{j=1}^{J}\left(\frac{\tilde{\tau}_{ji} F_{ji}}{Z^{\varepsilon_{ji}}} + \frac{\tilde{\tau}_{js} F_{js}}{Z^{\varepsilon_{js}}}\right) - \delta_z Z,$$

$$\dot{E}_j = \Omega_j\left(z_1, \left(E_j\right), Z, \left\{\bar{k}_j\right\}\right) \equiv \theta_{ji} F_{ji} + \theta_{js} F_{js} + \theta_j C_j - F_{je} - \bar{\theta}_j E_j + \Omega_j\left(\left(E_q\right)\right). \tag{A21}$$

Taking derivatives of equation (A19) with respect to t and applying (A21) to the resulted equation implies

$$\dot{\bar{k}}_1 = \frac{\partial \varphi}{\partial z_1}\dot{z}_1 + \Omega\frac{\partial \varphi}{\partial Z} + \sum_{j=1}^{J}\Omega_j\frac{\partial \varphi}{\partial E_j} + \sum_{j=2}^{J}\Phi_j\frac{\partial \varphi}{\partial \bar{k}_j}. \tag{A22}$$

Equaling the right-hand sizes of equations (A20) and (A22), we get

$$\dot{z}_1 = \Phi_1\left(z_1, \left(E_j\right), Z, \left\{\bar{k}_j\right\}\right) \equiv \left[\overline{\Phi}_1 - \sum_{j=1}^{J}\Omega_j\frac{\partial \varphi}{\partial E_j} - \sum_{j=2}^{J}\Phi_j\frac{\partial \varphi}{\partial \bar{k}_j}\right]\left(\frac{\partial \varphi}{\partial z}\right)^{-1}. \tag{A23}$$

Equations (A21) and (A23) implies the differential equations in the lemma. The procedure described previously implies the computational procedure in the lemma. In summary, we proved the lemma.

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